# M.A./M.Sc. (MATHEMATICS) SEMESTER –III

# (FOR STUDENTS OF SESSION 2024-25)

# **MACC-301: Functional Analysis**

#### **Course Outcomes:**

After the completion of the course, students are expected to have the ability to:

- 1. Apply Holder and Minkowski inequalities
- 2. Construct conjugate of a linear operator
- 3. Basic understanding of orthogonal and orthonormal sets
- 4. Construct complete orthonormal basis from a given linearly independent set
- 5. Describe the applications of spectral theorem

# Unit I

Banach Spaces- the definition and some examples, continuous linear transformations, The Hahn Banach theorem.

#### Unit II

The natural imbedding of N in N\*\*, the open mapping theorem, the conjugate of an operator.

# **Unit III**

Hilbert spaces- the definition and some simple properties, Orthogonal complements, orthogonal sets, the Conjugate space H\*.

# **Unit IV**

The adjoint of an operator, Self adjoint operators, normal and unitary operators, Projections.

#### Unit V

Finite dimensional spectral theory – Spectrum of an operator, the spectral theorem, uniqueness of spectral decomposition.

- 1. G. F. Simmons: Introduction to Topology & Modern Analysis (McGraw Hill).
- 2. V.B.Limaye: Functional Analysis, Wiley Eastern.

# **MACC-302: Structures on Almost Complex Manifolds**

#### **Course Outcomes:**

After the completion of the course, students are expected to have the ability to:

- 1. Explain the concept of even dimensional manifolds and their examples
- 2. Understand the concepts of contravariant almost analytic vector fields, covariant almost analytic vector fields, killing vector fields and their applications
- 3. Apply various concept of differential calculus to the settings of abstract set called manifold.
- 4. Show applications of Kahlermanifolds in superstring theory and in general relativity

#### Unit I

Almost complex manifolds, Nijenhuis tensor, contravariant and covariant analytic vector and their properties.

#### Unit II

Almost Hermite manifold, almost analytic vector fields curvature tensors, Linear connections and their properties.

## **Unit III**

Kahler manifolds, affine connections, curvature tensors, contravariant almost analytic vectors.

#### **Unit IV**

Nearly Kahler manifold, curvature identities, Curvature tensors, almost analytic vectors.

## Unit V

Almost Kahler manifolds, analytic vectors conformal transformations, curvature identities.

- 1. R.S. Mishra, Structures on a differentiable manifold and their applications, Chandrama Prakashan, Allahabad.
- 2. U. C. De., A. A. Shaikh, Complex manifolds and contact manifolds, Narosa Publishing House.

# MAEL -301A: Advanced Ordinary Differential Equations

## **Course Outcomes:**

After the completion of the course, students are expected to have the ability to:

- 1. Solve an ordinary differential equation, and characterize its solution
- 2. Understand the existence, uniqueness, and other properties of a solution of differential equations
- 3. Introduce the problems focusing on areas where mathematical ideas have had a major impact on human inquiry
- 4. Describe a physical problems into a differential equation
- 5. Show understanding of the differential equation (ordinary and partial both)
- 6. Analyze the structure of real-world problems and plan solutions strategies
- 7. Derive the properties of solutions of differential equations

## Unit I

Existence and uniqueness of theorem. Dependence of solutions on initial conditions. Dependence of solutions on parameters.

#### Unit II

General theory of Linear Differential Equations: Existence of solutions, Basic theory of the homogeneous linear system, The nonhomogeneous linear system, Linear independence and Fundamental Systems, properties of the homogeneous Linear equation, reduction of order, the non homogeneous equation, the adjoint vector equation, self-adjoint vector equation.

# Unit III

Matrix methods for homogeneous linear systems with constant coefficients: Two equation and two functions and two unknown functions, Example of the Matrix Methods. Eigenvalue problems, Green's matrix and its properties. Solutions of boundary value problem by Green's matrix.

## **Unit IV**

Sturm-Liouville System, Eigen functions, Bessel functions, Singular Sturm Liouville systems, Legendre functions boundary value problem for ordinary differential equation. Solution by Eigenfunction Expansion, and Green's functions, construction of Green's function for Ordinary differential equation.

## Unit V

Zeroes of solutions, Strums separation and comparison theorems. Oscillatory and nonoscillatory equations, Riccati's equation and its solution, Pruffer transformation, Lagrange's identity and Green's formula for second-order equation.

- 1. S.L. Ross: Differential Equations Blaisdell Publishing Company.
- 2. R.H.Cole: Ordinary Differential Equation by Appleton-Century-Crofts, New York.
- 3. S.L.Rose: Differential equations by Wiley Indian edition.

# **MAEL-301B**: Cryptography

## **Course Outcomes:**

After the completion of the course, students are expected to have the ability to:

- 1. Have a broad theoretical background in cryptography and information security
- 2. Apply number theory in cryptography
- 3. Have good knowledge of information security and cryptography which will help them to go in the field of research and industry

## Unit I

Secure communication, cryptographic applications, Symmetric cipher model, Substitution technique: Ceasar cipher, Mono-alphabatic cipher, Playfair cipher, Hill cipher, polyalphabatic cipher, one time pad, Transposition techniques, cryptanalysis of classical ciphers.

#### **Unit II**

Pseudorandom bit generator, Blum Blum Shub generator, linear feedback shift register sequences, Nonlinear feedback shift register, Stream cipher, Modern stream ciphers, RC4 stream cipher.

#### Unit III

Block cipher, Feistel cipher, simplified DES, Data encryption standard (DES), Advance encryption standard(AES), S-box design of DES and AES, Boolean functions, bent functions, construction of finite fields, modular polynomial arithmetic. Mode of operations, Attacks on block cipher.

#### **Unit IV**

Public key cryptosystem, RSA cryptosystem, RAS and factoring, Rabin encryption, Key management, Diffie Hellman key exchange, discrete logarithm, ElGamal encryption, Message integrity, cryptographic hash function, Hesh function based on block ciphers, Message authentication, Message authentication codes(MAC), digital signature, RSA digital signature scheme, ElGamal digital signature scheme.

# Unit V

Factoring: p-1 method, quadratic sieve, discrete logarithm: DL problem, Shanks Babystep Giant step algorithm, Pollard pho algorithm, Pohlig-Hellman algorithm, Elliptic curves, Elliptic curve arithmetic, Elliptic curve cryptography

- 1. Johannes A. Buchmann: Introduction to cryptography, Springer
- 2. William stallings: Cryptography and network security Principles and practices, Pearson education
- 3. Alferd J. Menezes, Paul C. Van Oorschot, Scott: A Handbook of applied cryptography,
  Vanstone, CRC press
- 4. Wade Trappe, Lawrance C.: Introduction to cryptography and coding theory, Washington
- 5. Behrouz A. Forouzan: Cryptography and network security, Tata McGraw-Hill
- 6. JosefPieprzyk, Thomoshardjono, Jennifer Seberry: Fundamentals of computer security, Springer

# **MAEL 302 A: Discrete Mathematics**

#### **Course Outcomes:**

After the completion of the course, students are expected to have the ability to:

- 1. Display familiarity with the mathematical models which are the integral part of the hardware and software of computer science
- 2. Elaborate and expand their understanding of the tools helpful in the implementation of circuit design, AI algorithms and compiler construction.

#### Unit I

Mathematical Logic, Statement calculus: Propositional logic, Logic operators or connectives, Well formed formula (wff), Construction of truth-table for a formula, Equivalence of formulas, Tautology, Contradiction argument, Valid argument, Proving validity by truth-table methods, Inference theory of statement calculus, Minimal sets of logic operators. Predicate calculus: Statement function and statement, Proving validity by the deduction method.

#### Unit II

Lattice theory and Boolean Algebra Lattice Theory: partial order relation, Partially ordered set, Totally ordered set, Hasse Diagrams, Lattice, Lattice as an algebraic system, Bounded lattice, Complemented lattice, Distributive lattice, Direct product, Lattice homomorphism

#### **Unit III**

Boolean algebra: Boolean functions, Principle of duality, Boolean function minimization, Sum of products and product of sums form, Normal forms, Conversion of normal forms into principal normal forms, Boolean function minimization, Logic circuits, Designing of logic circuits.

# **Unit IV**

Automata theory, Finite state automaton, Types of automaton, Deterministic finite state automaton, Non deterministic finite state automaton, Equivalence of NFA and DFA, Finite state Machines: Moore and Mealy machine.

## Unit V

Grammars and Languages, Regular language, Regular expression Equivalence of Regular language and finite state automaton, Grammar: Context-free and Context-sensitive grammar, LR Grammar: Construction of LR(0) parsing table, Construction of LR(1) parsing table, Decision algorithms for CFL.

- 1. Mendelson, Elliott: Introduction to Mathematical Logic, Chapman & Hall, 1997
- 2. John E. Hoprcroft, Rajeev Motwani, Jeffrey D. Ullman: Introduction to Aotomata Theory Languages and Computation, Pearson Education, 2000
- 3. Arnold B. H.: Logic and Boolean Algebra, Prentice Hall, 1962
- 4. K. H. Rosen: Discrete Mathematics and its applications, MGH 1999

# **MAEL-302B: Approximation Theory**

## **Course Outcomes:**

After the completion of the course, students are expected to have the ability to:

- 1. Understand the concept of interpolation processes, finite element methods etc.
- 2. Apply the knowledge of the theory, in research on this area.
- 3. Use famous Weierstrass Approximation theorem, first lacunary interpolation i.e., (0, 2) interpolation and Spline interpolation.

## Unit I

Different types of Approximations, Weierstrass Approximation Theorem, Monotone operators, Markoff inequality, Bernstein inequality.

## Unit II

Hermite and HF interpolation, Fejérs theorem for HF interpolation, Lobatto and Radau Quadrature formulas, (0, 2)-interpolation on the nodes of  $\pi_n(x)$ , existence, uniqueness.

## Unit III

Explicit representation of (0, 2) interpolation, Effectiveness of Least squares approximation as Uniform approximation.

#### **Unit IV**

Erdös-Túran Theorem, Jackson's theorems (I to V), Dini-Lipschitz theorem, Inverse of Jackson's theorem, Bernstein Theorems (I, II, III), Zygmund theorem.

# Unit V

Spline interpolation, existence, uniqueness, Explicit representation of cubic splines, certain extremal properties and uniform approximation.

- 1. T.J. Rivlin: An Introduction to the Approximation of Functions, Dover Publications, NY
- 2. E. W. Cheney: Introduction to Approximation Theory, McGraw-Hill Book Company.
- 3. A. Ralston: A First Course in Numerical Analysis, McGraw-Hill Book Company.

# **MAIN -301 Summer Internship**

# **MAIER-301A: Mathematics in Computer Designing**

#### **Course Outcomes:**

After the completion of the course, students are expected to have the ability to:

- 1. Show familiarity with the awareness and understanding of day-to-day applications of mathematics specially in computer sciences
- 2. Possess insights related to computer's hardware and software design

#### Unit I

Preliminary concepts of mathematical functions and relations.

# **Unit II**

Number theory, study of residues.

#### Unit III

Mathematical equations and matrices.

#### Unit IV

Applications in computer hardware design.

## Unit V

Elementary applications in software design.

- 2. Ellen Ullman: Close to the Machine
- 3. Eric Raymond: The Art of Unix Programming
- 4. Donald Knuth:- The Art of Computer Programming
- 5. Richard M. Stallman: Free Software, Free Society
- 6. Richard P. Gabriel: Patterns of Software
- 7. Richard P. Gabriel: Innovation Happens Elsewhere
- 8. Keith Curtis: After the Software Wars

# **MAIER-301B Astronomy & Astrophysics**

## **Course Outcomes:**

After the completion of the course, students are expected to have the ability to:

- 1. Use Celestial Coordinates system to specify the positions of stars, planets, satellites, galaxies and other celestial objects in three dimensional space
- 2. Explore the parent star Sun and its importance for sustaining life on the earth
- 3. Show familiarity with techniques to explore the solar surface temperature
- 4. Understand the solar atmosphere and its effect on the Earth and other planets
- 5. Understand the origin of the solar system
- 6. Explore the Terrestrial, Jovian & Dwarf planets, Comets and Meteoroids.
- 7. Describe the formation and evolution of stars which is a key research field for all major questions in astrophysics and cosmology
- 8. Deduce the implications for reionization and the early stages of chemical and photometric evolution of galaxies observed at high redshift
- 9. Illustrate Hertzsprung-Russell Diagram which is a graphical tool that astronomers use to classify stars according to their luminosity, spectral type, color, temperature and evolutionary stage
- 10. Plot the stars on the H-R diagram according to their temperatures, spectral classes, and luminosity by which astronomers can classify stars into their different types
- 11. Show understanding about the Binary star systems which are very important in astrophysics because calculations of their orbits allow the masses of their component stars to be directly determined, that in turn allows other stellar parameters, such as radius and density, to be indirectly estimated
- 12. Describe that the Galaxies are organized clusters of billions of stars, gas, dust, and matter in all other forms, all bound by the force of gravity
- 13. Provide details about their Milky Way, the galaxy containing our universe

# **Unit I: Celestial sphere**

Constellations and nomenclatures of stars, The cardinal points and circles on the celestial sphere, Coordinate system, Equatorial, Ecliptic system, Hour angle, Twilight, Spherical triangle, Polar triangle and related problems.

## **Unit II: The Sun and Planets**

**Sun:** Interior structure of the Sun, atmosphere, solar activity, sunspots and magnetic field, solar wind.

**Planets**: Study of Terrestrial planets, Jovian planets- their surface features & atmospheres. Tidal forces, Roche limit.

**Dwarf planets:** Definitions and locations.

Debris of the Solar system: Comets, Asteroids, Meteoroids.

## **Unit III: Stellar Structure and Evolution**

**Stars:** Magnitude scales, Colour index, Basic of star formation and evolution.

Hertzsprung-Russell (HR) diagram, Spectral classification, Energy generation of stars.

**Basics of degenerate remnants of stars**: White dwarfs, Neutron stars, Pulsars, Black Holes, Chandrasekhar limit.

**Stellar interiors**: Hydrostatic equilibrium, Pressure equation of state, Energy sources, Energy transport and convection.

# **Unit IV: High Energy Astrophysics**

**Observational tools:** Blackbody radiation, Specific intensity and flux density, Stellar parallax.

Formation & Structure of spectral lines, Radiative transfer.

**Radiative processes in Astrophysics:** Synchrotron emission, Energy loss and electron spectrum, Compton scattering, Bremstrahlung, Thermal bremstrahlung.

Binary stars: Classification, Accretion disks in binaries, Hulse-Taylor binary pulsar.

# Unit V: The Milky way galaxy and Galaxies beyond

What are Galaxies & its types, The Milky Way Galaxy: Structure, Mass, Size. Hubble's Classification, Formation of galaxies.

- 1. W. M. Smart, Textbook on Spherical Astronomy, Cambridge University Press.
- 2. I.Todhunter, Spherical trigonometry, The Macmillan company, London.
- 3. Modern Astrophysics, B. W. Carroll and D. A. Ostlie, Addison-Wesley Publishing Co.
- 4. Eric Chaisson & Steve Macmillan, Astronomy Today, Prentice Hall, New Jersey.
- 5. John D Fix, Astronomy-Journey to the Cosmic Frontier, Mosby, New York.
- 6. Introductory Astronomy & Astrophysics, M. Zeilik and S. A. Gregory, 4th Edition, Saunders College Publishing.
- 7. Theoretical Astrophysics, Vol. I: Astrophysical Processes, T. Padmanabhan, Cambridge University Press
- 8. Theoretical Astrophysics, Vol. II: Stars and Stellar Systems, T. Padmanabhan, Cambridge University Press.
- 9. The Physical Universe: An Introduction to Astronomy, F. Shu, Mill Valley: University Science Books.
- 10. Textbook of Astronomy and Astrophysics with Elements of Cosmology, V. B. Bhatia, Pub-New Delhi, Narosa Publishing House.
- 11. The New Cosmos, A. Unsold and B. Baschek, New York: Springer Verlag.
- 12. The Physical Universe: An Introduction to Astronomy, F. Shu, Mill Valley: University Science Books.
- 13. Introduction to Cosmology, J. V. Narlikar, 3rd edition, Cambridge University Press.
- 14. Structure Formation in the Universe, T. Padmanabhan, Cambridge University Press.

#### **SEMESTER-IV**

# MACC - 401 :Advanced Linear Algebra

#### **Course outcome:**

After the completion of the course, students are expected to have the ability to:

- 1. Find the minimal polynomial of an operator.
- 2. Compute Jordan canonical form of an operator and apply it to practical problems.
- 3. Demonstrate Cholesky. polar, singular value, QR and LU decompositions of a matrix.

## Unit I

Review of characteristic values and characteristic vectors of linear operators, minimal polynomial, Primary decomposition theorem, Cayley Hamilton theorem.

## **Unit II**

Diagonalisable and triangulable operators, simultaneous diagonalization and triangularization.

## **Unit III**

Jordan canonical form, Jordan chain, Jordan block, Existence of Jordan canonical form, Applications of the Jordan form.

# **Unit IV**

Unitary similarity, Schur decomposition, Method of least squares, Cholesky decomposition, Hadamard inequality

# Unit V

Matrix decompositions – Polar decomposition, QR decomposition, LU decomposition, Singular value decomposition.

- 1. K. Hoffman and R. Kunze: Linear Algebra, Prentice Hall
- 2. V. Sahai and V. Bist: Linear Algebra, Narosa
- 3. H. Helson: Linear Algebra, Hindustan Book Agency

# **MAEL-401A: Advanced Partial Differential Equations**

## **Course Outcomes:**

After the completion of the course, students are expected to have the ability to:

- 1. Describe a physical problems in the form of differential equations
- 2. Solve the partial differential equation, and get characterized its solutions
- 3. Provide a sufficient knowledge to analyse the structure of real-world problems and plan solutions strategies
- 4. Understand the properties of solutions of differential equations
- 5. Understanding existence, uniqueness, and other properties of a solution of differential equations

#### Unit I

Introduction, basic concept and definition, classification of second order linear equation and method of characteristics, canonical form, Equations with constant coefficients, Supper position principle. Method of separation of variables.

#### **Unit II**

Boundary Value Problems, Maximum and Minimum Principles, Uniqueness and Stability theorem, Dirichlet problem for a Circle, Dirichlet Problem for a Circular annulus, Neumann problem for a Circle, Dirichlet problem for a Rectangular, Dirichlet problem involving Poisson equation.

#### **Unit III**

Fourier Transforms - Definition and properties, Fourier transform of some elementary functions, convolution theorem, Application of Fourier transforms to solve partial differential equations. Green's functions and boundary value problems.

## **Unit IV**

The Cauchy problem: The Cauchy problem, Cauchy-Kowalewsky Theorem, Hadmard example, Cauchy problem for homogeneous wave equations, Initial value problem, The Cauchy problem for Non-homogeneous wave equation., The vibration string problem, Existence and uniqueness solution of the vibrating problem.

#### Unit V

Duhamel's Principle: Wave Equations, Heat Conduction equations, solution of Heat Equation in two and three dimensions (Cartesian, polar and spherical coordinate system) in terms of Bessel's and Legendre's Functions.

- 1. TynMyint-U: Partial Differential Equations of Mathematical Physics, Elsevier Publication
- 2. I.N. Sneddon: Elements of Partial Differential Equations, McGraw-Hill, 1988

# MAEL- 401B: Almost Contact Manifolds & F-Structure Manifolds

## **Course Outcomes:**

After the completion of the course, students are expected to have the ability to:

- 1. Explain the concept of odd dimensional manifolds and their examples
- 2. Understand the concepts of Lie derivative, Sasakian manifolds, K-contact Riemannian manifolds, properties of curvature on these manifolds
- 3. Understand the concept of different submanifolds
- 4. Apply the course knowledge in super-gravity, superstring theory and in general relativity
- 5. Apply for advanced studies / research in this area

## Unit I

Almost contact manifold, Lie derivative, affinity almost Co-Symplectic manifold.

#### Unit II

Almost Grayan manifold, almost Sasakian manifold, K-contact Riemannian manifold, Properties of curvature on these manifolds, Almost contact 3-structure, Para contact structure.

#### **Unit III**

Sasakian manifolds, quasi-Sasakian manifold, 3-structure metric manifold, Kenmotsu manifold, trans-Sasakian manifold and their properties.

## Unit IV

Co-symplectic structure, F- structure manifold and their properties.

#### Unit V

Submanifolds of almost Hermite manifolds and Kahler manifolds, Almost Grayansubmanifolds.

- 1. R. S. Mishra: Structures on a differentiable manifold and their applications, ChandramaPrakashan, Allahabad
- 2. U. C. De., A. A. Shaikh: Complex manifolds and contact manifolds, Narosa Publishing house

# MAEL-402A: Graph Theory

#### **Course Outcomes:**

After the completion of the course, students are expected to have the ability to:

- 1. Illustrate multiple applications of Graphs in real life
- 2. Make models which are used in designing networks of highways, railways and airways
- 3. Outline data structures. which are useful in computer science.

#### Unit I

Graph and its terminology, Directed and undirected graph, Multi graph, Simple graph, Complete graph, Weighted graph, Planar and non-planar graph, Regular graph, Graph isomorphism and homeomorphism, Euler's formula, Statement and applications of Kuratowski's theorem

## Unit II

Representing graphs in computer system, Coloring of graph. Graph connectivity, Konigsberg bridge problem, Euleria path and Elerian circuit, Hamiltonian path and Hamiltonian circuit.

#### Unit III

Study of Shortest path and shortest distance, Dijkstra's algorithm, Paths between the vertices, Path matrix, Warshall's algorithm, cut point, bridge, cut sets and connectivity, Menger's theorem.

# **Unit IV**

Tree and related terminology, spanning tree, Finding minimum spanning tree by Kruskal's algorithm and Prim's algorithm, inorder, preorder, and postorder tree traversals, Binary tree, Expression trees and reverse polish notation (RPN), RPN evaluation by stack.

#### Unit V

Flow network, Feasible flows, Multiple sources and multiple sinks, cut sets in flow network, Relation between flows and cuts, Max flow problem, Max flow min-cut theorem, Matching, Covering, Application of networks in Operations Research –PERT.

- 1. Harary, Addison: Graph Theory Wesley 1969
- 2. D. B. West: Introduction to Graph Theory, Prentice Hall 1996.
- 3. Jonathan Gross and Jay Yellan: Graph Theory and its Applications, CRC 1998.

# MAEL-402B: Wavelet Theory

## **Course Outcomes:**

After the completion of the course, students are expected to have the ability to:

- 1. Make plans for higher level research in this area and get funded by the funding agencies
- 2. Learn about the mathematical analysis involved in WA
- 3. Know the short comings in the Fourier analysis and how Wavelets help in overcoming those difficulties
- 4. Review Fourier transformation, continuous wavelet transform, multi-resolution analysis and algebraic constructions, included in the course

#### **UNIT I**

Fourier and inverse Fourier transforms, Convolution and delta function, Fourier transform of Square integrable functions, Continuous Wavelet Transform: The Heisenberg uncertainty principle, the Shannon sampling theorem, Definition and examples of continuous wavelet transforms,

#### **UNIT II**

A Plancherel formula, Inversion formulas, the kernel functions, Decay of wavelet transform, Frames: Geometrical considerations, Notion of frames.

# **Unit III**

Discrete wavelet transforms, signal decomposition (analysis), relation with filter banks, signal reconstruction.

# **UNIT IV**

Multi resolution analysis, axiomatic description, the scaling function, construction of Fourier domain.

# **UNIT V**

Orthonormal wavelets with compact support: the basic idea, Algebraic constructions, binary interpolation, spline wavelets.

- 1. Christian Blatter: Wavelets A Premier, AK Peters, 2002.
- 2. C.K. Chui: An Introduction to Wavelets, Academic press.
- 3. Daubechies: Ten Lectures on Wavelets, SIAM, Philadelphia.

# **MAMT-401: Master Thesis**

# MAIRA-401A: Mathematical Biology

## **Course Outcomes:**

After the completion of the course, students are expected to have the ability to:

- 1. Identify and explain the role of mathematics in the process of modelling in the field of Natural and Social Sciences.
- 2. Apply the various growth models of population
- 3. Explain and apply the models of harvesting and Mutualism

#### Unit I

Continuous population Model for single species, Exponential population growth model, continuous population growth model: Malthes model for population growth , General population growth model, Qualitative Analysis: Equilibrium points, Stability Analysis, logistic population growth model and their qualitative analysis, logistic growth model for non isolated population .

#### Unit II

Insect Out break model: Spruce Budworm ,Continuous Single species population model with Delays: Introduction, General Delay model and Qualitative Analysis, Logistic model with time delay effects, equilibrium points & stability Analysis.

## **Unit III**

Harvesting a single Natural population: Harvesting in Delayed recruitment models: Constant effort Harvesting, constant yield harvesting, population model with Age Distribution, Simple Discrete population model.

# **Unit IV**

Continuous Models for Interacting Population: Interaction between species: two species models, definition of stability, community matrix approach ,Qualitative behaviour of community matrix, Competition: Lotka-Volterra models, Extension to Lotka-Volterra models, competition in field experiments, competition for space, Models for Mutualism.

#### Unit V

Predator: Prey interaction: Lotka-Volterra models, dynamic of simple Lotka-Volterra models, Role of density dependent in the Prey, Classic laboratory experiment on predator ,predation in natural system. Some predator- prey models .

- 1. J.D.Murrey: Mathematical Biology
- 2. Alan Hasting: Population Biology Concepts and Models, Springer
- 3. J.Mazumdar: An introduction to Mathematical Physiology & Biology
- 4. Fred Brauer, Carlos Castillo- Chivez: Mathematical Models in Population Biology and Epidemiology

# **MAIRA-401B: Astro-Biology**

#### **Course Outcomes:**

After the completion of the course, students are expected to have the ability to:

- 1. Describe the origin and evolution of life in the cosmos: What is life? how did it form? and where is it?
- 2. Display understanding of relatedness and integration of disciplines to Astro-biology.
- 3. Show understanding of comets and their impact on our solar system
- 4. Study Quantum chemistry and its tools which provide the theoretical information regarding to the formation of molecules that plays an important role for the origin of life in the Interstellar medium, planetary and Cometary atmospheres.
- 5. Explain what kind of environment would be needed to allow life to begin by creating hypothetical conditions present on the early Earth.

# **Unit I: Introduction to Astrobiology**

- What is Astrobiology: A brief description, applications and importance.
- **Prospects for life elsewhere in the Solar System**: Sites with the possibility of liquid water and prebiotic molecules.
- Organic molecules in Meteorites.
- Space Missions for exploring the possibility of life.

# Unit II: Comets-The unexpected visitors and Astrobiology

- Comets: Structure, composition, classification & origin.
- Potential sources of prebiotic molecules for the early earth.
- Organic molecules and volatiles in comets.
- Cometary missions.

# **Unit III: From Interstellar Molecules to Astrobiology**

- **Interstellar Medium:** Definition and composition of ISM.
- The relationship of Interstellar molecules to the origin of life.
- Formation of interstellar molecules: In diffuse clouds, cold dark clouds, hot molecular cores and grain surface.
- Organic molecules in circumstellar envelopes.
- Prebiotic reactions and RNA world.

# **Unit IV: Basic Quantum Chemistry**

- Quantum Astrochemistry: Definition, approximation & application of Quantum chemistry to search the origin of life.
- **Methods of Computational Quantum Chemistry:** Application of computational techniques and Quantum chemical methods etc.
- Introduction and usages of Gaussian Programme package.

## **Unit V – Extra-terrestrial Intelligence**

- **Origin of Life on Earth** Theories about origin of life, Urey-Miller experiment.
- Extrasolar Planets The Circum stellar habitable zone, The Inner limit and outer limit of HZ, Continuous habitable zone, Galactic habitable zone.
- Methods to detect Extrasolar planets.

- 1. C.N. Banwell: Fundamentals of Molecular Spectroscopy
- 2. Baidyanath Basu: An Introduction to Astrophysics
- 3. Exploring Chemistry with Electronic Structure methods James B. Foresman&Aeleen Frisch.
- 4. Astrobiology: Future Perspectives- Pascale Ehrenfreund, William Irvine.
- 5. Lectures in Astrobiology Vol I Muriel Gargaud, Bernard Barbier, Herve Martin & Jacques Reisse.
- 6. Lectures in Astrobiology Vol II- Muriel Gargaud, BernardBarbier, Herve Martin & Jacques Reisse.
- 7. Chemical evolution and Origin of Life- Horst Rauchfuss.
- 8. Comets and the Origin and Evolution of Life- Paul J. Thomas, Christopher F. Chyba & Christopher P. Mckay.
- 9. Life in Universe- Joseph Seckbach, Julian Chela Flores, Tobias Owen and Francois Raulin.
- 10. Eric Chaisson& Steve Macmillion, Astronomy Today, Prentice Hall, New Jersey.John D Fix, Astronomy-Journey to the Cosmic Frontier, Mosby, New York.

# **MAIRA-401C: Special Functions**

#### **Course Outcomes:**

After the completion of the course, students are expected to have the ability to:

- 1. Perform operations with orthogonal polynomials with their differential equations along with the corresponding recurrence relations.
- 2. Understand the concept of Hermite Polynomial, Laguerre Polynomial, Jacobi Polynomial etc with its properties.
- 3. Demonstrate their understanding of how physical phenomena are modeled using special functions.
- 4. Understand how special function is useful in differential equation.

## Unit I

Orthogonal Polynomials, Simple Set of Polynomials, Orthogonality, Equivalent Condition for Orthogonality, Zeros of Orthogonal Polynomials, Recurrence Relation, Christoffel Darboux Formula.

#### **Unit II**

Definition of Hermite Polynomial  $H_n(x)$ , Pure Recurrence Relations, Differential Recurrence Relation, Rodrigue's Relation, Generating Functions, Orthogonality, integral Representation of Hermite Polynomial, Differential Equation and its Solution.

## **Unit III**

Laguerre Polynomial  $L_n(x)$ , Definition, Differential Equation and its Solution, Generating Function, Recurrence Relation, Rodrigues Formula.

#### **Unit IV**

Jacobi's Polynomial, Differential Equation, Bateman's Generating Function, Rodrigue's Formula, Recurrence Relation.

#### Unit V

Chebychev Polynomial, Independent Solution of Chebychev Polynomial, Expansion of  $T_n(x)$  and  $U_n(x)$ , Generating Functions, Recurrence Relations, Orthogonality.

- 1. Special Functions: E. D. Rainville
- 2. Orthogonal Polynomials: G. Szego